Thermoviscoelastic Analysis of Axisymmetric Solid Propellant Grains

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Theme

THIS paper presents a technique developed for the analysis of stress concentration in axisymmetric viscoelastic solids with a moving inner boundary. Using White's[1] procedure of viscoelastic analysis, the technique is then used to analyze a case-bonded grain, in which a number of axisymmetric slots are spaced at intervals along the longitudinal directions. Since the grain material constants are highly temperature sensitive, a general thermoviscoelastic analysis is formulated. However, the combination of low thermal conductivity coefficients and the burning rates typical of solid propellants leads to the assumption that the high flame temperature at a burning surface does not change the bulk temperature distribution in the propellant.

Contents

Starting from the principle of virtual work and using linear polynomial shape functions for the triangular axisymmetric elements TRIAX-3,[2] the final incremental stiffness equations for an element are derived as usual, following the procedure given in Ref. 1. The relaxation modulus is expressed in a Prony series[3] so that the memory load vector can be calculated from a recurrence relation which involves strains from the two previous time steps. Now, since the grain boundary moves with time during burning, at each instant of time, element matrices have to be generated for the new configuration. An automatic mesh generation routine has been developed for this purpose. In this mesh generation procedure, the total number of elements n is maintained the same at any time. At any particular instant, knowing the profile of burning surface, the new configuration is fixed. The domain is now divided into n number of elements and the coordinate data for the nodes constituting the elements are generated. Since the total number of elements is the same, this is equivalent to the spatial displacement of a particular element between two time intervals. This displacement can be made arbitrarily small by choosing small time steps. Hence, it can be assumed that the stresses, strains and memory loads are carried over to the new displaced position of the elements.

In order to study the reliability of the computer program developed, a test problem of a thick viscoelastic cylinder subjected to internal pressure was solved in which a close agreement between the exact and present solutions could be obtained.

Figure 1 illustrates a representative slice of the whole grain where the movement of inner boundary during burning is shown by dotted lines, and the variation of the parallel shift R of the burning surface with respect to time is shown by a graph. The relaxation modulus $G(t)$ is considered in the form $G(t) = 0.1 + 17.9 \exp(-t/1.11) + 15.3(-t/132)$, and the bulk modulus, $K = 2752$ kg/cm^2. The results shown in Figs. 1 and 2 indicate that, due to the change of geometry of the burning surface, there is a definite trend towards relaxation not only of stresses but also strains. Alternatively, if one considers the grain geometry to be fixed with the same loading

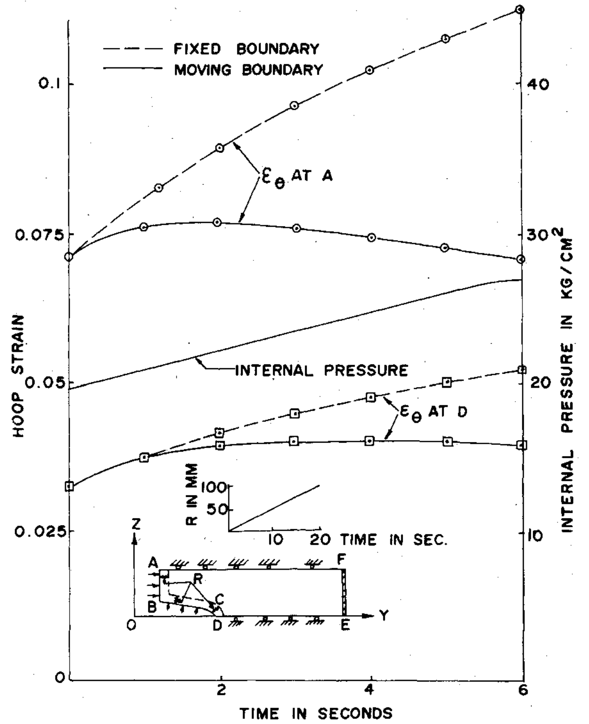

Fig. 1 Variation of hoop strains with time.

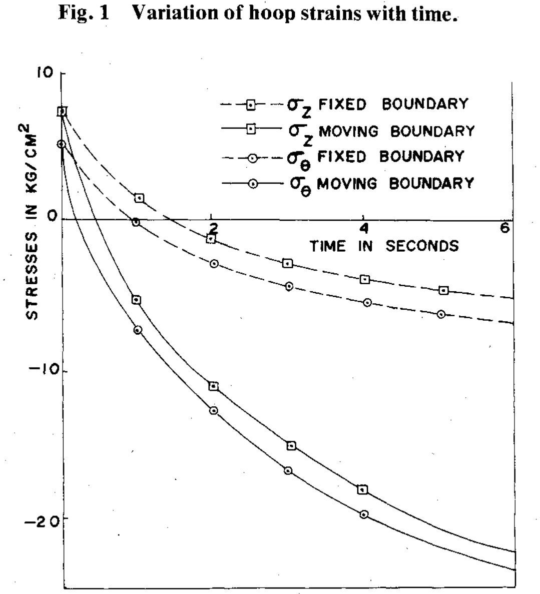

Fig. 2 Variation of stresses at D with time.

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Index category: Structural Static Analysis.

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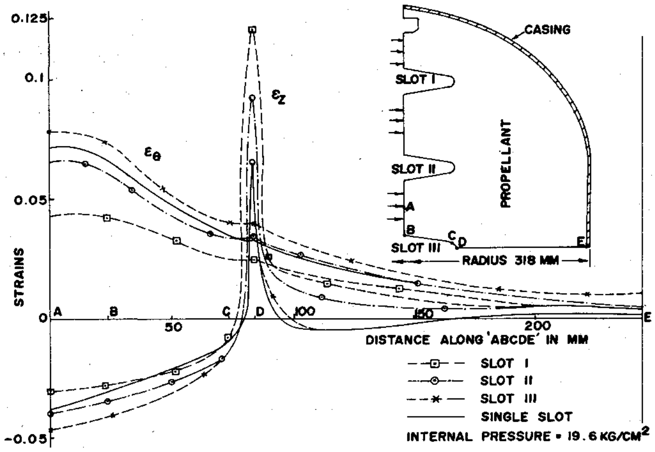

Fig. 3 Hoop and axial strains along 'ABCDE' for the different slots of the full grain.

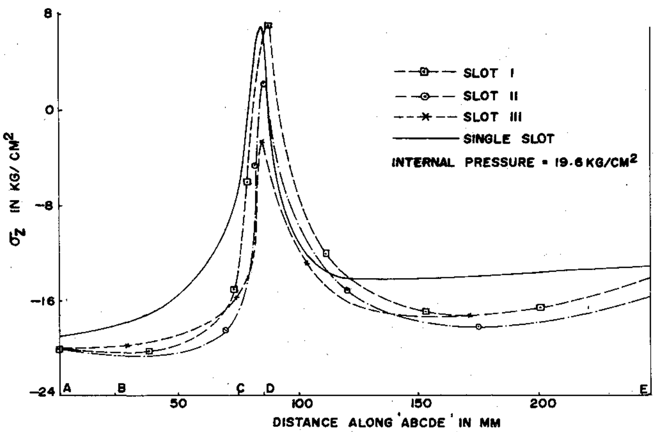

Fig. 4 Axial stresses along 'ABCDE' for the different slots of the full grain.

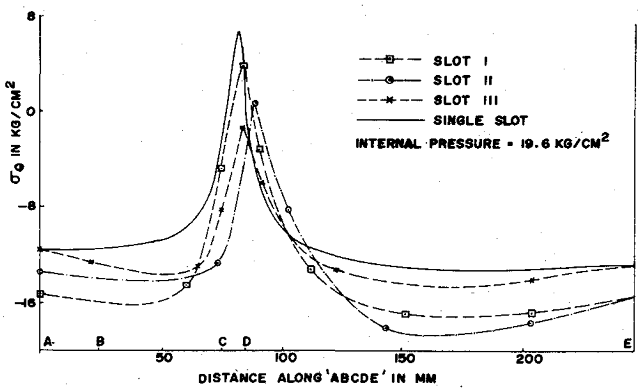

Fig. 5 Hoop stresses along 'ABCDE' for the different slots of the full grain.

conditions, the result would indicate a stress relaxation and an increase in the hoop and axial strain components with time.

The error involved in the analysis of a representative slice of the full grain is then studied by analyzing one half of the whole grain (Fig. 3) for internal pressure at time $t=0$. The results presented in Figs. 3-5 exhibit the variations obtained along the slots, if one uses the single-slot solution instead of going to a tedious full grain analysis. The present results are obtained assuming two terms of Prony series; the analysis also can be performed without any further complexity by including any number of terms required for actual representation of the material properties of the propellant.

References

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[2]Zienkiewicz, I.C., *The Finite Element Method in Engineering Science*, Mc Graw-Hill, London, 1971.

[3]Zak, A.R., "Structural Analysis of Realistic Solid Propellant Materials", *Journal of Spacecraft and Rockets*, Vol. 5, March 1968, pp. 270-275.